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INCLUSIVE DETERMINATIONS OF $|V_{ub}|$ AND $|V_{cb}|$

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ABSTRACT

In this talk I review the status of our ability to extract the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ from inclusive semileptonic decays. I focus on model independent determinations of these parameters and discuss the expected theoretical uncertainties.

1 Introduction

The magnitudes of the Cabbibo, Kobayashi, Maskawa (CKM) matrix elements V_{ub} and V_{cb} are two of the parameters of the standard model which can be determined at current experimental facilities producing B mesons. Semileptonic decays of B mesons mediated by the weak decay of a b quark to either an up or a charm quark are an ideal way to perform these measurements, since the part

of the process involving the leptonic final states can be calculated perturbatively. The theoretical calculations required can be split into two parts. First, the decay rate of the b quark to either an up or a charm quark is required, and second the hadronic effects which bind these quarks into the observed hadrons in the experiment have to be dealt with.

The perturbative expressions for the $b \rightarrow u\ell\bar{\nu}$ decay is known to order α_s^2 ¹⁾, while the $b \rightarrow c\ell\bar{\nu}$ decay rate is currently known to order $\alpha_s^2\beta_0$ ²⁾, where β_0 is the one loop coefficient of the QCD beta function. The hadronization effects can not be calculated perturbatively and is governed by long distance physics. There are two distinct ways to extract the CKM from decays of B mesons. One can use exclusive decays to a well defined hadronic final state, such as D or D^* mesons for $b \rightarrow c$ transitions, or π or ρ mesons for the measurement of $|V_{ub}|$. All non-perturbative physics is then encoded in the hadronic form factors. For D and D^* mesons heavy quark effective theory (HQET) ³⁾ can be used to obtain the form factor at leading order in an expansion in $1/m_{b,c}$ at the zero recoil point ⁴⁾, and because of Luke's theorem ⁵⁾ corrections are absent at order $1/m_{b,c}$. For the decay to an up quark HQET is not applicable, and the relevant form factors have to be determined using other non-perturbative methods, such as lattice QCD ⁶⁾ or QCD sumrules ⁷⁾. Recently there has also been progress using the soft collinear effective theory ⁸⁾ to determine the required form factors from experiment.

An alternative approach is to use decays to inclusive final states, which include all final states containing either an up or a charm quark. Decays to such inclusive final states can be calculated using the operator product expansion (OPE), which states that at leading order in $1/m_b$ the inclusive decay is identical to the perturbatively calculable parton level decay. Corrections are given by matrix elements of local operators, which are suppressed by powers in $1/m_b$. By determining enough of these matrix elements the CKM parameters V_{ub} and V_{cb} can be determined with high accuracy. I review the recent progress on inclusive determinations of V_{ub} and V_{cb} in this talk.

2 Inclusive determination of V_{ub}

The inclusive decay rate $B \rightarrow X_u\ell\bar{\nu}$ is directly proportional to $|V_{ub}|^2$ and can be calculated reliably and with small uncertainties using the operator product expansion (OPE). Unfortunately, the ~ 100 times background from $B \rightarrow X_c\ell\bar{\nu}$

makes the measurement of the totally inclusive rate an almost impossible task. Several cuts have been proposed in order to reject the $b \rightarrow c$ background, however care has to be taken to ensure that the decay rate in the restricted region of phase space can still be predicted reliably theoretically. The proposed cuts are

1. Cut on the lepton energy $E_\ell > (m_B^2 - m_D^2)/(2m_B)$
2. Cut on the hadronic invariant mass $m_X < m_D$ ⁹⁾
3. Cut on the leptonic invariant mass $q^2 > (m_B - m_D)^2$ ¹⁰⁾
4. Cut on light cone component of the hadronic momentum $P_+ < m_D^2/m_B$ ¹¹⁾
5. Combined lepton-hadron invariant mass cut ¹²⁾

While the cut on the energy of the charged lepton is easiest to implement experimentally, it has the largest theoretical uncertainties. This is due to the fact that only $\sim 10\%$ of the $b \rightarrow u$ events survive this cut, amplifying any higher order, uncalculated terms drastically. Thus, it is not useful for a precision determination of $|V_{ub}|$, although it can be used as a check for consistency.

The remaining four cuts each have their advantages and disadvantages, and it remains to be seen which will yield the individually smallest uncertainty on $|V_{ub}|$ ultimately. To illustrate the effect of these four phase space cuts, we show the allowed phase space of the $B \rightarrow X_u \ell \bar{\nu}$ transition, in terms of two light cone projections of the hadronic four-momentum,

$$\begin{aligned} P_+ &= n \cdot P = E - |\vec{P}| \\ P_- &= n \cdot P = E + |\vec{P}|. \end{aligned} \quad (1)$$

The projections satisfy $P_+ P_- = P^2$ and thus it is obvious that the boundaries of phase space are

$$m_\pi^2/P_- < P_+ < P_- < m_B \quad (2)$$

The resulting phase space diagram is shown in Fig. 1. Also displayed in a rough distribution of the events obtained from a toy Monte Carlo simulation. While this distribution should not be viewed as a sound theoretical prediction, it qualitatively helps to understand the phase space better. The region of

phase space occupied by the $b \rightarrow c$ background is given by $P_+ P_- > m_D^2$ and is indicated by the gray area.

The region satisfying $P_+ \ll P_-$, denoted by the ellipse in Fig. 1, is called the shape function region. The decay rate in the presence of cuts which include this region contain higher dimensional operators contributing at order $(P_+ \Lambda_{\text{QCD}}/P_-^2)^n$. This fraction becomes order unity and all these terms have to be resummed to all orders into an unknown function, called the shape function¹³⁾. This function is a universal property of the B meson, and can be measured in other B decays, such as the radiative decay $B \rightarrow X_s \gamma$. Note that it is not simply related to the b quark mass and the kinetic energy of the b quark as is often assumed¹⁴⁾. In fact, at leading order in both α_s and Λ_{QCD}/m_b , the shape of the photon energy spectrum is precisely given by this light cone distribution function. At order $1/m_b$ several new subleading shape functions enter¹⁵⁾, which are at present completely unknown. Thus, even with perfect knowledge of the photon energy spectrum in $B \rightarrow X_s \gamma$ the uncertainties in regions of phase space which include the shape function region of order Λ_{QCD}/m_b .

The regions of phase space surviving the four cuts are also illustrated in Fig. 1. On the left we show the $m_X < m_D$ and $P_+ < m_D^2/m_B$ cuts, which both include the shape function region, while on the right we show the $q^2 > (m_B - m_D)^2$ and the combined hadron-lepton invariant mass cut, which do not include the shape function region. It is clear that the cut on the hadronic invariant mass $m_X < m_D$ ⁹⁾ is optimal in the sense that it keeps all events which are not accessible by $b \rightarrow c \ell \bar{\nu}$ transitions. It has been estimated that $\sim 80\%$ of the $b \rightarrow u$ events survive this cut. Uncertainties from subleading shape functions are of order Λ_{QCD}/m_b , however they have recently been estimated to be at the few percent level¹⁶⁾. Precise knowledge of the shape function is however still required to achieve an uncertainty on $|V_{ub}|$ below the 10% level.

The situation is similar for the cut on the light cone momentum P_+ , which also includes the shape function region. While this cut includes slightly less phase space, it has been argued that the relationship between the shape function and the differential rate of $B \rightarrow X_s \gamma$ is slightly simpler for this cut than for the m_X cut described above¹¹⁾. The resulting uncertainties on $|V_{ub}|$ are expected to be at the same order as for the m_X cut.

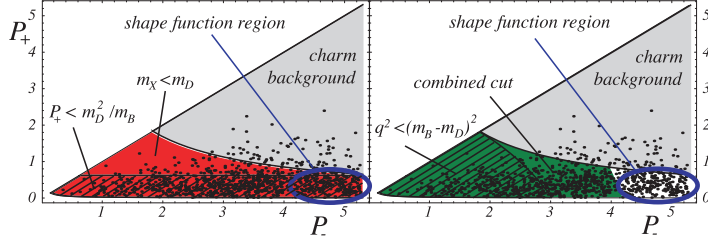


Figure 1: The dalitz plot in the q^2/s_H and q^2/E_ℓ plane. In both plots the gray area denotes the area contaminated by $b \rightarrow c$ events. The left plot shows the $m_X < m_D$ and $P_+ < M_D^2/m_B$ cuts, while the right hand plot shows the $q^2 > (m_B - m_D)^2$ and the combined $q^2 - m_X$ cut. Also shown in both plots is the shape function region.

The situation is qualitatively different for the remaining two cuts, which involve a cut on the leptonic invariant mass. Since a lepton invariant mass cut removes the shape function region, the decay rate in the presence of these cuts can be calculated using the standard OPE in an expansion in local operators, but the expansion is in powers of $1/m_c$ rather than $1/m_b$ ¹⁷⁾. For the pure q^2 cut, where $q^2 > (m_B - m_D)^2$, the fraction of events surviving the cut is estimated to be about $(17 \pm 3)\%$ ¹⁰⁾. This gives an uncertainty on $|V_{ub}|$ at the 10% level.

The final cut discussed here is a combined cut on both the hadronic and the leptonic invariant mass. The idea here is to use the cut on m_X to remove the charm background, and the cut on q^2 to keep the sensitivity on the shape small. The ideal combination of cuts remains to be determined in a detailed experimental study, but using the combined cuts $m_X < m_D$ GeV, $q^2 > 6$ GeV² one finds the fraction of surviving events to be $(45 \pm 5)\%$ ¹²⁾. Since the decay rate is proportional to $|V_{ub}|^2$, this allows for a determination of $|V_{ub}|$ with uncertainties well below the 10% level.

To summarize, there are currently five types of cuts to eliminate the charm background proposed in the literature. While a cut on the lepton energy is easiest to measure, it has by far the largest theoretical problems. A cut on the leptonic invariant mass alone also leads to relatively large theoretical uncertainties and will probably not yield a measurement of $|V_{ub}|$ with uncertainties below the 10% level. The remaining three cuts all can yield a determination of this CKM matrix element with uncertainties considerably below the 10% level,

and all of them should be used together for a precision measurement of $|V_{ub}|$.

3 Inclusive determination of V_{cb}

Inclusive semileptonic B decays can be calculated using an operator product expansion (OPE). This leads to a simultaneous expansion in powers of the strong coupling constant $\alpha_s(m_b)$ and inverse powers of the heavy b quark mass. At leading order in this expansion this reproduces the parton model result

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} (1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \log \rho) , \quad (3)$$

where $\rho = m_c^2/m_b^2$, and nonperturbative corrections are suppressed by at least two powers of m_b . The state of the art is to use theoretical predictions to order $\alpha_s^2 \beta_0^{-2}$ in the perturbative expansion, to order $\Lambda_{\text{QCD}}^3/m_b^3$ ¹⁸⁾ in the non-perturbative power expansion and to order $\alpha_s \Lambda_{\text{QCD}}/m_b$ in the mixed terms. Here β_0 is the one loop coefficient of the QCD beta function $\beta_0 = 25/3$ for $n_f = 4$ light quark flavors. There are no non-perturbative contributions at order $1/m_b$ and thus the inclusive rate can be written schematically as

$$\begin{aligned} \Gamma^{b \rightarrow c} = \Gamma_0 & \left\{ 1 + A \left[\frac{\alpha_s}{\pi} \right] + B \left[\left(\frac{\alpha_s}{\pi} \right)^2 \beta_0 \right] + 0 \left[\frac{\Lambda}{m_b} \right] + C \left[\frac{\Lambda^2}{m_b^2} \right] \right. \\ & \left. + D \left[\frac{\Lambda^3}{m_b^3} \right] + E \left[\frac{\alpha_s}{\pi} \frac{\Lambda}{m_b} \right] + \mathcal{O} \left(\alpha_s^2, \frac{\Lambda^4}{m_b^4}, \alpha_s \frac{\Lambda^2}{m_b^2} \right) \right\} , \quad (4) \end{aligned}$$

The coefficients $A - E$ depend on the quark masses $m_{(c,b)}$. At order $\Lambda_{\text{QCD}}^2/m_b^2$ there are two matrix elements ($\lambda_{1,2}$) parametrizing the non-perturbative physics, while at order $\Lambda_{\text{QCD}}^3/m_b^3$ there are six additional matrix elements ($\rho_{1,2}, \mathcal{T}_{1-4}$).

The total inclusive branching fraction for B decays is currently measured with uncertainties around 2%. To predict this branching ratio with comparable precision requires detailed knowledge of the value of the matrix elements $\lambda_{1,2}$ and even some rough knowledge of the matrix elements at order $\Lambda_{\text{QCD}}^3/m_b^3$. The best way to determine these parameters is to use the semileptonic data itself. Many differential decay spectra have been measured, and moments of these spectra have been calculated to the same accuracy as the total branching ratio itself ¹⁹⁾. A global fit to all experimental data is able to test how well the OPE is able to describe the inclusive observables ²⁰⁾.

The mass of the b -quark which naturally appears in the OPE calculations is the pole masse. It has been long known that using these pole masses gives rise to a poorly behaved perturbative expansion, due to the presence of a renormalon. There are several threshold mass definitions, which do not contain a renormalon, called $1S$ mass ²¹⁾ PS mass ²²⁾ , and kinetic mass ²³⁾.

The c quark can be treated as a heavy quark. This allows one to compute the $D^{(*)}$ meson masses as an expansion in powers of Λ_{QCD}/m_c . The observed $B-D$ mass splitting can be used to determine $m_b - m_c$. Since the computations are performed to $\Lambda_{\text{QCD}}^3/m_c^3$, this introduces errors of fractional order $\Lambda_{\text{QCD}}^4/m_c^4$ in m_c , which gives fractional errors of order $\Lambda_{\text{QCD}}^4/(m_b^2 m_c^2)$ in the inclusive B decay rates, since charm mass effects first enter at order m_c^2/m_b^2 . This is the procedure used in Ref. ²⁰⁾. An alternative approach is to avoid using the $1/m_c$ expansion for the charm quark ²⁴⁾. In this case heavy quark effective theory (HQET) can no longer be used for the c quark system, and there are no constraints on m_c from the D and D^* meson masses. At the same time, it is not necessary to expand heavy meson states in an expansion in $1/m_{b,c}$, so that the time-ordered products \mathcal{T}_{1-4} can be dropped. The number of parameters is the same whether or not one expands in $1/m_c$.

Currently, there are 75 pieces of data available combining moments of the hadronic invariant mass spectrum and the lepton energy spectrum of inclusive measured of semileptonic decays and the photon energy spectrum in $B \rightarrow X_s \gamma$ by BABAR ²⁶⁾, BELLE ²⁷⁾, CDF ²⁸⁾, CLEO ²⁹⁾ and DELPHI ³⁰⁾ together with moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ measured by BABAR, BELLE and CLEO. These observables can all be predicted using the same OPE and have been calculated in all of the mass schemes discussed above and depend on 7 parameters. A global fit to all these 75 observables was performed in ²⁵⁾. This allowed to extract the value of $|V_{cb}|$ simultaneously with the non-perturbative parameters of the OPE. It was shown that all schemes give consistent values for $|V_{cb}|$, m_b and the matrix elements appearing at order $1/m_b^2$. In Table 1 we show the results of the fits in the $1S$ and the kinetic scheme. One can see that the two schemes give consistent results, with the uncertainties in the $1S$ scheme being slightly smaller than the ones in the kinetic scheme.

| Scheme | $ V_{cb} \times 10^3$ | $m_b^{1S} [\text{GeV}]$ | $\lambda_1 [\text{GeV}^2]$ |
|---------------------------|------------------------|--------------------------|----------------------------|
| $1S_{\text{exp}}$ | 42.1 ± 0.6 | 4.68 ± 0.04 | -0.23 ± 0.06 |
| kin_{exp} | $42.2 \pm 0.4 \pm 0.4$ | $4.67 \pm 0.04 \pm 0.02$ | $-0.17 \pm 0.06 \pm 0.06$ |

Table 1: Fit results for $|V_{cb}|$, m_b and λ_1 in the $1S$ and kin schemes, where m_c is obtained from the $B - D$ mass splitting.

4 Conclusions

In this talk I reviewed the current status of determining the magnitude of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ from inclusive semileptonic B meson decays. For $B \rightarrow X_c \ell \bar{\nu}$, the operator product expansion has been calculated to order $1/m_b^3$, with a total of 6 parameters in addition to $|V_{cb}|$ appearing at that order. These 6 parameters can be determined in a fit to precision measurements of inclusive decay spectra and one finds $|V_{cb}| = (42.1 \pm 0.6) \times 10^{-3}$. Also obtained in the fit is the value of the b -quark mass and the parameter λ_1 , which are shown in Table 1.

To measure $|V_{ub}|$ from the inclusive decay $B \rightarrow X_u \ell \bar{\nu}$ one has to deal with the large background from $b \rightarrow c$ transitions. Imposing kinematic cuts to suppress this background tends to destroy the convergence of the OPE. Several cuts have been presented which allow to suppress this background experimentally, and in the future it should be possible to determine the value of $|V_{ub}|$ with uncertainties well below the 10% level.

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